TABLE 21

STEP IMPEDANCES OF 21-ELEMENT FILTERS

N = 21 VSWR = 1.05

BW	0.60	0.70	0.80	1.00	1.20	1.40
L(DB)	221.98	192.60	166.78	122.49	84.72	51.05
1	1.715	1.554	1.406	1.242	1.148	1.091
2	0.3684	0.4399	0.5134	0.6598	0.7928	0.8989
3	3.549	2.982	2.548	1.930	1.523	1.262
4	0.2729	0.3227	0.3750	0.4900	0.6237	0.7747
5	3.985	3.380	2.919	2.256	1.788	1.433
6	0.2587	0.3047	0.3522	0.4535	0.5679	0.7052
7	4.101	3.484	3.016	2.349	1.884	1.527
8	0.2545	0.2994	0.3457	0.4433	0.5511	0.6768
9	4.141	3.520	3.050	2.380	1.917	1.566
10	0.2529	0.2979	0.3436	0.4400	0.5458	0.6671
11	4.2	3.51	3.056	2.388	1.926	1.576

N = 21 VSWR = 1.10

BW	0.60	0.70	0.80	1.00	1.20	1.40
L(DB)	227.80	198.42	172.60	128.31	90.54	56.87
1	1.956	1.732	1.570	1.359	1.232	1.153
2	0.3472	0.4139	0.4830	0.6244	0.7608	0.8792
3	3.791	3.196	2.741	2.090	1.652	1.358
4	0.2725	0.3216	0.3729	0.4841	0.6117	0.7577
5	4.148	3.521	3.045	2.363	1.885	1.522
6	0.2616	0.3078	0.3555	0.4566	0.5690	0.7007
7	4.238	3.603	3.122	2.435	1.961	1.600
8	0.2585	0.3038	0.3505	0.4489	0.5564	0.6793
9	4.2	3.630	3.147	2.460	1.986	1.631
10	0.27	0.304	0.3491	0.4464	0.5524	0.6721
11			3.14	2.466	1.993	1.638

N = 21 VSWR = 1.20

BW	0.60	0.70	0.80	1.00	1.20	1.40
L(DB)	233.44	204.06	178.24	133.95	96.18	62.51
1	2.301	2.018	1.812	1.537	1.366	1.255
2	0.3367	0.4004	0.4666	0.6044	0.7433	0.8726
3	4.106	3.470	2.985	2.291	1.819	1.490
4	0.2788	0.3286	0.3803	0.4913	0.6165	0.7599
5	4.392	3.731	3.230	2.514	2.016	1.640
6	0.2704	0.3181	0.3672	0.4705	0.5841	0.7146
7	4.462	3.795	3.290	2.571	2.076	1.703
8	0.268	0.3151	0.3634	0.4647	0.5748	0.6986
9	4.4	3.81	3.309	2.589	2.095	1.726
10		0.316	0.3624	0.4629	0.5718	0.6934
11			3.30	2.593	2.100	1.732

N = 21 VSWR = 1.50

BW	0.60	0.70	0.80	1.00	1.20	1.40
L(DB)	240.43	211.05	185.23	140.94	103.17	69.50
1	3.013	2.637	2.325	1.926	1.671	1.498
2	0.3477	0.4121	0.4789	0.6195	0.7663	0.9124
3	4.760	4.133	3.481	2.691	2.150	1.762
4	0.3054	0.3595	0.4154	0.5340	0.6655	0.8145
5	4.977	4.231	3.667	2.862	2.307	1.891
6	0.2904	0.3520	0.4060	0.5193	0.6424	0.7810
7	5.028	4.278	3.710	2.903	2.350	1.938
8	0.298	0.3499	0.4033	0.5151	0.6359	0.7698
9		4.29	3.724	2.917	2.364	1.955
10		0.35	0.403	0.5138	0.6338	0.7661
11			3.71	2.921	2.368	1.959

N = 21 VSWR = 2.00

BW	0.60	0.70	0.80	1.00	1.20	1.40
L(DB)	245.20	215.83	190.00	145.71	107.94	74.27
1	3.861	3.337	2.946	2.408	2.059	1.816
2	0.3845	0.4548	0.5275	0.6809	0.8432	1.010
3	5.601	4.752	4.108	3.188	2.557	2.100
4	0.3490	0.4106	0.4740	0.6079	0.7549	0.9200
5	5.786	4.921	4.266	3.335	2.695	2.218
6	0.3440	0.4043	0.4662	0.5957	0.7358	0.8917
7	5.82	4.960	4.303	3.369	2.731	2.258
8	0.35	0.4029	0.4640	0.5923	0.7304	0.8825
9		4.9	4.31	3.381	2.743	2.271
10		0.42	0.465	0.591	0.729	0.880
11			4.2	3.382	2.745	2.275

Synthesis of Symmetrical TEM-Mode Directional Couplers

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Abstract—Exact synthesis procedures are derived for symmetrical three-section and five-section TEM-mode directional couplers. These synthesis procedures are based on the equivalence between the theory of directional couplers and stepped quarter-wavelength filters as previously described by Levy and Young. Explicit formulas for the parameters of three-section couplers are presented.

A realizable insertion-loss function is derived for the five-section coupler resulting in an equal-ripple response. Although this function has an equal-ripple characteristic, it is not expressible in terms of Chebyshev polynomials. Results obtained for the five-section coupler show considerable improvement in bandwidth over a three-section coupler. For example, a five-section coupler of -3 ± 0.5 dB has a bandwidth of 9.6:1 as compared with 5.8:1 for a three-section coupler of -3 ± 0.5 dB.

A five-section coupler of -10 ± 0.5 dB was designed on this theoretical basis for the 555–4000 Mc band, and the measured performance shows good agreement with the theoretical coupling response, yielding a minimum directivity of 18.0 dB at 3.7 Gc.

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Introduction

The TEM-mode directional coupler has been the subject of a number of current papers. Shimizu and Jones [1] have obtained, by numerical substitution, the solution to the symmetrical three-section coupler for a limited number of average couplings.

Levy [2] has recently derived an exact synthesis procedure for asymmetrical multisection couplers. However, Levy's synthesis procedure is not readily applicable to symmetrical couplers because their symmetry imposes additional restrictions on the insertion-loss function of the primary line.

Young and Cristal [3] have independently compiled tables of designs for three-, five-, seven-, and nine-section symmetrical couplers utilizing a similar synthesis approach. Our results on the three- and five-section couplers are in full agreement with theirs.

The equivalence between the theory of directional couplers, as established by Young [4] and Levy [2], and that of stepped quarter-wavelength filters is shown in Fig. 1. The coupling to arms 2 and 4 are, respectively, the reflection and transmission coefficients of cascaded transmission lines of electrical length θ and even-mode impedances $Z_{oe1}Z_{oe2}, \cdots, Z_{oen}$ terminated by lines of unit impedance. The normalized even- and odd-mode impedances are related by

$$Z_{oe1}Z_{oo1} = Z_{oe2}Z_{oo2} = \cdots = Z_{oen}Z_{oon} = 1 \qquad (1)$$

if the coupler is to retain the ideal VSWR and isolation property [2]. On the basis of this equivalance, the transfer matrix for the lossless n-section filter is given by

$$\begin{bmatrix} A_n & jB_n \\ jC_n & D_n \end{bmatrix} = \prod_{r=1}^{n} \begin{bmatrix} \cos\theta & jZ_r \sin\theta \\ j/Z_r \sin\theta & \cos\theta \end{bmatrix} \qquad (2)$$

and the insertion loss to arm 4 is

$$L = 1 + 1/4(A_n - D_n)^2 + 1/4(B_n - C_n)^2. \qquad (3)$$

But for a symmetrical two-port network

$$A_n = D_n. \qquad (4)$$

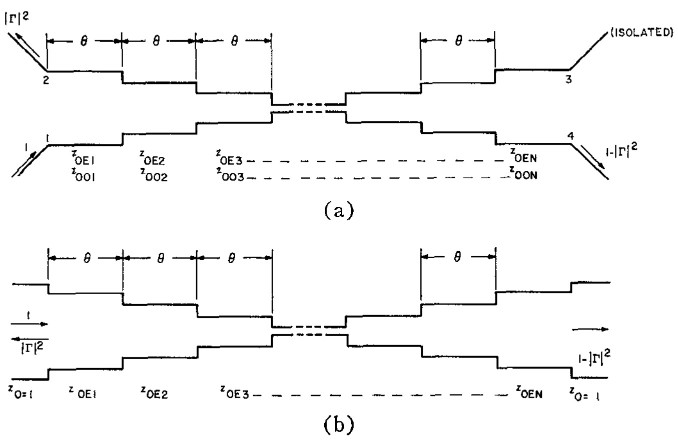

Fig. 1. (a) n-section symmetrical coupler. (b) Equivalent n-stepped quarter-wavelength filter.

Then (3) becomes

$$L = 1 + 1/2(B_n - C_n)^2. \qquad (5)$$

It can readily be shown [5], by performing the matrix multiplication in (2) and using (5), that

$$L = 1 + P_n^2(\sin\theta) \qquad (6)$$

where $P_n(\sin\theta)$ is an odd polynomial in $\sin\theta$ of degree n. Only an odd number of sections is considered here since a symmetrical structure necessarily contains an odd number of sections. Introducing Richard's [6] frequency transformation $t=j \tan\theta$, the insertion-loss function in (6) becomes

$$L(t) = 1 + P_n^2(jt/\sqrt{1-t^2}) \qquad (7)$$

and the reflection coefficient is given by

$$\Gamma(t)\Gamma(-t) = \frac{L-1}{L} = \frac{P_n^2(jt/\sqrt{1-t^2})}{1 + P_n^2(jt/\sqrt{1-t^2})}. \qquad (8)$$

As stated by Seidel and Rosen [7], the necessary and sufficient conditions for realizing a symmetrical stepped-impedance filter of n equal-line-length sections are the following:

1) The insertion-loss function must have the form of (7), where P_n is an odd polynomial for n odd only.
2) $\Gamma(t)$ and $\Gamma(-t)$ must have identical zeros.

The second condition implies that the zeros of the reflection coefficient have symmetry with respect to the j axis in the complex t plane. Thus, based on the realizability conditions given, the synthesis of symmetrical couplers consists mainly of two steps:

1) Find the optimum polynomial $P_n(\sin\theta)$ for an equal-ripple approximation to the coupling response.
2) Find the odd- and even-mode impedances of the n lines of electrical length θ ($\theta=90°$ at the center frequency) utilizing exact synthesis procedures.

Young [4] has indicated that the insertion-loss function of symmetrical couplers cannot be expressed in terms of known Chebyshev polynomials, as is the case of asymmetrical couplers. However, as shown in the following section, a third-degree Chebyshev polynomial with a properly restricted argument can still be used to obtain an optimum insertion-loss function for three-section symmetrical couplers. It is also found that when this technique is applied to higher-degree Chebyshev polynomials, the resulting coupling response is not optimum in the sense that it does not provide maximum bandwidth, and thus new polynomials have to be found for directional couplers having more than three sections. Crystal and Young [3] have obtained such equal-ripple polynomials by an iterative method. The purpose of this paper is to present exaxt synthesis procedures for three- and five-section symmetrical couplers. It should be emphasized that the formulas derived here for the odd-and even-mode impedances of three-section couplers are presented in close form and as such should prove very helpful in computational work.

Synthesis of Symmetrical Three-Section Couplers

The synthesis of symmetrical three-section directional couplers is accomplished by letting

$$P_3(\sin\theta) = hT_3(\sin\theta/s) \tag{9}$$

in (6), under the restriction

$$0 < (\sin\theta/s) < \sqrt{3}/2 \tag{10}$$

for $0<\theta<\pi$, where $\sqrt{3}/2$ is the positive real root of the third-degree Chebyshev polynomials. The constant parameters h and s are defined later in this section. In essence, the purpose of introducing restriction (10) is to prevent the occurrence of the real roots of T_3 $(\sin\theta/s)$ in the pass band of the coupler. Construction details of the resulting insertion-loss function are shown in Fig. 2. Having imposed (10), the insertion-loss function to be realized takes the form

$$L = 1 + h^2T_3^2(\sin\theta/s) \tag{11}$$

and the square of the reflection coefficient is given by

$$|\Gamma(\sin\theta/s)|^2 = \frac{L-1}{L} = \frac{h^2T_3^2(\sin\theta/s)}{1+h^2T_3^2(\sin\theta/s)}. \tag{12}$$

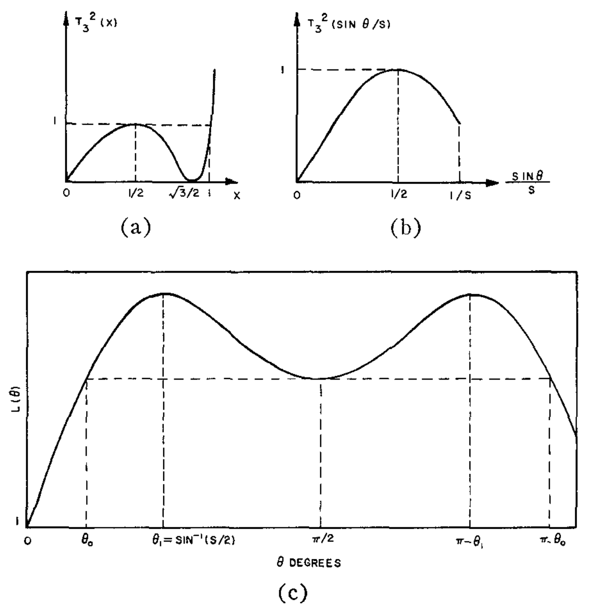

Fig. 2. Diagrams showing construction of insertion-loss function L for a three-section coupler.

The following synthesis procedure is similar to that used by Levy [2] to synthesize asymmetrical couplers, but it is presented here for completeness.

The first step in the synthesis is to determine $\Gamma(t)$ by obtaining the roots of the numerator and the denominator in (12). The roots of the numerator in (12) are given by

$$\cosh\left[3\cosh^{-1}\left(\frac{\sin\theta_r}{s}\right)\right] = 0$$

or

$$3\cosh^{-1}\left(\frac{\sin\theta_r}{s}\right) = j(2r-1)\frac{\pi}{2} \qquad (r = 1, 2, 3).$$

Then

$$\sin\theta_r = s\left[\cos\left(\frac{2r-1}{3}\right)\frac{\pi}{2}\right] \qquad (r = 1, 2, 3) \tag{13}$$

and since $t = j\tan\theta$, the complex roots are

$$t_r^2 = \frac{s^2\cos^2\left[\left(\frac{2r-1}{3}\right)\frac{\pi}{2}\right]}{s^2\cos^2\left[\left(\frac{2r-1}{3}\right)\frac{\pi}{2}\right]-1}. \tag{14}$$

Equation (14) gives the following three roots:

$$t_1^2 = t_3^2 = \frac{3s^2}{3s^2-4} \tag{15}$$

$$t_2^2 = 0. \tag{16}$$

Similarly, the roots of the denominator in (12) are

$$t_r'^2 = \frac{s^2\cosh^2\left[\frac{J}{3}+j\left(\frac{2r-1}{3}\right)\frac{\pi}{2}\right]}{s^2\cosh^2\left[\frac{J}{3}+j\left(\frac{2r-1}{3}\right)\frac{\pi}{2}\right]-1} \qquad (r = 1, 2, 3) \tag{17}$$

where

$$J = \sinh^{-1}(1/h). \tag{18}$$

Then

$$\Gamma(t)\Gamma(-t) = K^2\frac{t^2(t^2-t_1^2)^2}{(t^2-t_1'^2)(t^2-t_2'^2)(t^2-t_3'^2)} \tag{19}$$

where

$$K^2 = \frac{h^2T_3^2\left(\frac{1}{s}\right)}{1+h^2T_3^2\left(\frac{1}{s}\right)}. \tag{20}$$

As previously stated, $\Gamma(t)$ must have the same zeros as $\Gamma(t)$, and in addition its poles must lie in the left-half plane if $\Gamma(t)$ is to be analytical in the right-half plane.

Selecting the poles and zeros in this manner leads to the formula

$$\Gamma(t) = K\frac{t(t^2 - t_1^2)}{(t + t_2')[t^2 + (t_1' + t_1'^*)t + |t_1'|^2]} \quad (21)$$

where the real parts of t_1', t_2' are all positive. The expressions for the coefficients of the quadratic in t appearing in the denominator of (21) are given by

$$T_1' = t_1' + t_1'^* = \sqrt{2\left[1 + \left(1 - \frac{A}{A^2 + B^2}\right)|t_1'|^2\right]}\ |t_1'| \quad (22)$$

$$|t_1'|^2 = \frac{s^2(\sinh^2 J/3 + 3/4)}{\sqrt{s^4(\sinh^2 J/3 + 3/4)^2 - s^2(\sinh^2 J/3 + 3/2) + 1}} \quad (23)$$

where

$$A = \frac{s^2}{2}(1 + \tfrac{1}{2}\cosh 2J/3)$$

$$B = \frac{s^2\sqrt{3}}{4}\sinh 2J/3.$$

Using (21), the input impedance $Z(t)$ is derived from the formula

$$Z(t) = \frac{1 + \Gamma(t)}{1 - \Gamma(t)} = \frac{P_3(t)}{Q_3(t)} \quad (24)$$

where $P_3(t)$ and $Q_3(t)$ are positive real polynomials of third degree.

The next step in the synthesis procedure is to find the values of the impedances of two lines (because of the symmetry, the first and third lines have the same impedance) of electrical length θ, terminated by lines of unit impedance (Fig. 1) which give this value of $Z(t)$. The transfer matrix of a single line section of impedance Z is

$$\begin{bmatrix} \cos\theta & jZ\sin\theta \\ j1/Z\sin\theta & \cos\theta \end{bmatrix} = \frac{1}{\sqrt{1 - t^2}}\begin{bmatrix} 1 & Zt \\ t/Z & 1 \end{bmatrix} \quad (25)$$

where $t = j\tan\theta$. Hence the overall transfer matrix for three sections is

$$\frac{1}{(1 - t^2)^{3/2}}\prod_{r=1}^{3}\begin{bmatrix} 1 & Z_r t \\ t/Z_r & 1 \end{bmatrix} = \frac{1}{(1 - t^2)^{3/2}}\begin{bmatrix} A(t) & B(t) \\ C(t) & D(t) \end{bmatrix} \quad (26)$$

where in this case $A(t) = D(t)$ because of the symmetry of the network. The input impedance is given by

$$Z(t) = \frac{A(t) + B(t)}{A(t) + C(t)} \quad (27)$$

where $A(t)$ is even and $B(t)$, $C(t)$ are odd polynomials in t. The overall matrix in (26) can also be expressed by

$$\frac{1}{(1 - t^2)^{3/2}}\begin{bmatrix} EP_3(t) & OP_3(t) \\ OQ_3(t) & EQ_3(t) \end{bmatrix} \quad (28)$$

where E and O refer to the even and odd parts of the polynomials $P_3(t)$ and $Q_3(t)$. Combining (26) and (28) leads to

$$\begin{bmatrix} EP_3(t) & OP_3(t) \\ OQ_3(t) & EQ_3(t) \end{bmatrix} = \prod_{r=1}^{3}\begin{bmatrix} 1 & Z_r t \\ t/Z_r & 1 \end{bmatrix}. \quad (29)$$

To obtain the value of Z_1 the overall matrix (29) is premultiplied by the inverse matrix

$$\frac{1}{1 - t^2}\begin{bmatrix} 1 & -Z_1 t \\ -t/Z_1 & 1 \end{bmatrix} \quad (30)$$

and then the condition that all elements of the resulting matrix must be divisible by $(1 - t^2)$ is applied. This process is repeated in order to determine Z_2. Following this procedure, the input impedance $Z(t)$ is obtained from (21), that is,

$$Z(t) = \frac{P_3(t)}{Q_3(t)} = \frac{ct^3 + at^2 + dt + b}{et^3 + at^2 + ft + b} \quad (31)$$

where

$$a = T_1' + t_2' \quad (32a)$$

$$b = t_2'|t_1'|^2 \quad (32b)$$

$$c = 1 + K \quad (32c)$$

$$d = |t_1'|^2 + t_2'T_1' - Kt_1^2 \quad (32d)$$

$$e = 1 - K \quad (32e)$$

$$f = |t_1'|^2 + t_2'T_1' + Kt_1^2 \quad (32f)$$

and where

$$EP_3 = EQ_3(t) = at^2 + b \quad (33a)$$

$$OP_3 = ct^3 + dt \quad (33b)$$

$$OQ_3 = et^3 + ft. \quad (33c)$$

Then the overall transfer matrix becomes

$$\frac{1}{(1 - t^2)^{3/2}}\begin{bmatrix} at^2 + b & ct^3 + dt \\ et^3 + ft & at^2 + b \end{bmatrix}. \quad (34)$$

Premultiplying (34) by (30) gives the matrix

$$\frac{1}{(1-t^2)^{5/2}}\begin{bmatrix} 1 & -Z_1 t \\ -t/Z_1 & 1 \end{bmatrix}\begin{bmatrix} at^2+b & ct^3+dt \\ et^2+ft & at^2+b \end{bmatrix}$$

$$= \frac{1}{(1-t^2)^{5/2}}\begin{bmatrix} -eZ_1t^4+(a-Z_1c)t^2+b & (c-aZ_1)t^3+(d-Z_1b)t \\ \left(e-\dfrac{a}{Z_1}\right)t^3+\left(f-\dfrac{b}{Z_1}\right)t & -\dfrac{c}{Z_1}t^4+\left(a-\dfrac{d}{Z_1}\right)t^2+b \end{bmatrix}. \quad (35)$$

Since all elements of (35) are divisible by $(1-t^2)$, Z_1 is given by

$$Z_1 = Z_3 = \frac{a+b}{e+f} = \frac{c+d}{a+b}. \quad (36)$$

Matrix (35) now reduces to

$$\frac{1}{(1-t^2)^{3/2}}\begin{bmatrix} -eZ_1t^2-b & (c-aZ_1)t \\ \left(e-\dfrac{a}{Z_1}\right)t & -\dfrac{c}{Z_1}t^2-b \end{bmatrix}. \quad (37)$$

Repeating this process by premultiplying (37) by

$$\frac{1}{(1-t^2)}\begin{bmatrix} 1 & -Z_2t \\ -\dfrac{t}{Z_2} & 1 \end{bmatrix} \quad (38)$$

shows that

$$Z_2 = \frac{eZ_1+b}{\dfrac{a}{Z_1}-e} = \frac{aZ_1-c}{\dfrac{c}{Z_1}+b}. \quad (39)$$

It can also be shown that

$$Z_2^2 = \frac{1-K}{1+K}Z_1^4. \quad (40)$$

Equations (36) and (39) give the normalized odd-mode impedances of the three-section coupler as functions of h and s which can be determined given the mean coupling C (dB) and the coupling deviation R (dB). As shown in Fig. 2, the bandwidth (in terms of the electrical length of one coupling section) extends from θ_0 to $\pi-\theta_0$. θ_0 is determined as follows:

The coupling in dB to arm 2 (Fig. 1) is given by the expression

$$\eta = 10\log_{10}|\Gamma|^2 = 10\log_{10}\left[\frac{h^2T_3^2(\sin\theta/s)}{1+h^2T_3^2(\sin\theta/s)}\right] \quad (41)$$

from which

$$\eta_{\max} = 10\log_{10}\left[\frac{h^2}{1+h^2}\right] \quad (41a)$$

$$\eta_{\min} = 10\log_{10}\left[\frac{h^2T_3^2(\sin\theta_0/s)}{1+h^2T_3^2(\sin\theta_0/s)}\right]$$

$$= 10\log_{10}\left[\frac{h^2T_3^2(1/s)}{1+h^2T_3^2(1/s)}\right] \quad (42b)$$

where

$\eta_{\max}$ = maximum coupling in dB for $\theta=\sin^{-1}(s/2)$ $\theta=\pi-\sin^{-1}(s/2)$ (Fig. 2).

$\eta_{\min}$ = minimum coupling in dB for $\theta=\pi/2$, θ_0, and $(\pi-\theta_0)$.

By definition, the mean coupling C (dB) and the coupling deviation R (dB) are given by

$$C = \tfrac{1}{2}(\eta_{\max}+\eta_{\min}) \quad (43a)$$

$$R = \tfrac{1}{2}(\eta_{\max}-\eta_{\min}) \quad (43b)$$

or

$$\eta_{\max} = C+R \quad (44a)$$

$$\eta_{\min} = C-R. \quad (44b)$$

Combining (42) and (43) gives the following important formulas:

$$h = \left(\frac{\log_{10}^{-1}[C+R)/10]}{1-\log_{10}^{-1}[(C+R)/10]}\right)^{1/2} \quad (45a)$$

$$s = \sec(\pi/3-\phi) \quad (45b)$$

$$\sin\theta_0 = s\cos(\pi/3+\phi) \quad (45c)$$

where

$$\phi = \tfrac{1}{3}\cos^{-1} \cdot\left(\frac{[\log_{10}^{-1}(C-R)/10][1-\log_{10}^{-1}(C+R)/10]}{[\log_{10}^{-1}(C+R)/10][1-\log_{10}^{-1}(C-R)/10]}\right)^{1/2}. \quad (45d)$$

Then the bandwidth ratio $(\pi-\theta_0)/\theta_0$ can easily be calculated using (45). This bandwidth ratio is shown in Fig. 3 as a function of coupling deviation for the −3-dB and −10-dB couplers. As an example, consider the following coupler design:

Coupling: −3 dB ± 0.5 dB.

From (45) it is found that

$$h = 1.134, \quad s = 1.480, \quad (\pi-\theta_0)/\theta_0 = 5.82{:}1.$$

Substituting these values into (21)–(39), one obtains

$$Z_3 = Z_1 = 0.7692, \quad Z_2 = 0.2614.$$

Z_1 and Z_2 are normalized odd-mode impedances and are related to even-mode impedances by (1). Renor-

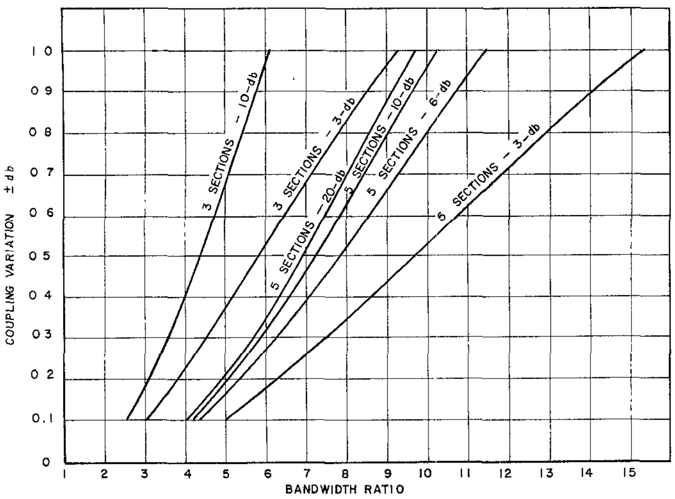

Fig. 3. Coupling variation of various directional couplers.

malizing these impedances to 50 Ω gives the following ohmic values for the three-section coupler:

Section	1	2	3
Z_{oe}	65.00	191.28	65.00
Z_{oo}	38.46	13.07	38.46

Furthermore, based on the foregoing equations, the odd- and even-mode characteristic impedances and coupling response of a symmetrical three-section −3-dB coupler, having equal-ripple variations of ±0.1, ±0.2, ±0.3, ±0.4, ±0.5 dB, were also calculated and found to be in excellent agreement with previously published results [1]. To avoid repetition, these results are not included in this paper.

Synthesis of Symmetrical Five-Section Couplers

If the technique of restricting the argument of Chebyschev polynomials is extended to the general case of multisection symmetrical couplers, it can readily be shown that very little improvement in bandwidth is achieved. In particular, when a fifth-degree Chebyshev polynomial is used to design a five-section coupler of −3±0.5 dB, the resulting theoretical bandwidth ratio is 6.20:1 as compared with 5.82:1 for a three-section coupler having the same average coupling and coupling deviation. The reason that this method does not provide an optimum response for $n>3$ is that it restricts the number of equal ripples to only one.

It is known that a symmetrical five-section coupler can have an equal-ripple response with two ripples in the pass band [8]. To obtain such a response, the following polynomial is introduced:

$$P_s(\sin\theta) = \sin^5\theta - 2\alpha^2\sin^3\theta + (\alpha^4+\beta^4)\sin\theta \quad (46)$$

whose roots are

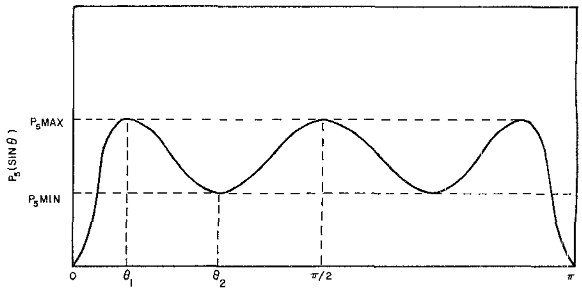

Fig. 4. Equal-ripple variation of function P_5 (sin θ).

$$\sin\theta_3 = 0 \quad (47a)$$

$$\sin^2\theta_4 = \alpha^2 + j\beta^2 \quad (47b)$$

$$\sin^2\theta_5 = \alpha^2 - j\beta^2 \quad (47c)$$

where α and β are real quantities. The purpose of introducing polynomial (46) with complex roots is to avoid any real zeros in the pass band of the coupler since it is necessary that $L>1$ in the pass band.

Subsituting (46) into (6) gives

$$L = 1 + h^2[\sin^5\theta - 2\alpha^2\sin^3\theta + (\alpha^4+\beta^4)\sin\theta]^2. \quad (48)$$

Since an equal-ripple response is desired, P_5 (sin θ) must vary, as in Fig. 4. The curve has three equal maxima and two equal minima. By differentiating P_5 (sin θ) with respect to θ, its maxima and minima can be shown to occur at

$$\sin^2\theta_1 = \frac{3\alpha^2 - 2\sqrt{\alpha^4 - 5\beta^4/4}}{5} \quad (49a)$$

$$\sin^2\theta_2 = \frac{3\alpha^2 + 2\sqrt{\alpha^4 - 5\beta^4/4}}{5} \quad (49b)$$

and for equal maxima and minima it is necessary that the following equations be satisfied simultaneously:

$$hP_5(\sin\theta_1) = k_1 \quad (50a)$$

$$hP_5(1) = k_1 \quad (50b)$$

$$hP_5(\sin\theta_2) = k_2 \quad (50c)$$

where

$$k_1 = \sqrt{\eta_{\max}/(1-\eta_{\max})} \quad (51a)$$

$$k_2 = \sqrt{\eta_{\min}/(1-\eta_{\min})}. \quad (51b)$$

Equations (50) constitute a system of nonlinear equations, with h, α, and β as the unknown quantities. Because of the complexity involved in solving this system of equations, its solution was obtained with the aid of an electronic computer. The results are given in Table I. The resulting polynomials have an equal-ripple variation, but they cannot be expressed in terms of known Chebyshev polynomaials.

Now, using (48), the reflection coefficient of the equivalent stepped impedance filter is given by the formula

$$|\Gamma|^2 = \frac{h^2 \sin^2\theta[\sin^4\theta - 2\alpha^2\sin^2\theta + (\alpha^4+\beta^4)]^2}{1 + h^2\sin^2\theta[\sin^4\theta - 2\alpha^2\sin^2\theta + (\alpha^4+\beta^4)]^2}. \tag{52}$$

The roots of the numerator in (52) are given by (47), and in the complex frequency plane t they become

$$t_1^2 = \frac{\alpha^2 - j\beta^2}{(\alpha^2-1) - j\beta^2} \tag{53a}$$

$$t_2^2 = \frac{\alpha^2 + j\beta^2}{(\alpha^2-1) + j\beta^2} \tag{53b}$$

$$t_3^2 = 0. \tag{53c}$$

The roots of the denominator in (52) are not so easily calculable because of the need to solve a fifth-degree equation. For this reason they were obtained with the

TABLE I

SOLUTION TO (50) FOR VARIOUS DEGREES OF COUPLING FOR THE SYMMETRICAL FIVE-SECTION COUPLERS

	$C=-3$ dB			$C=-6$ dB		
R(dB)	h	α	β	h	α	β
±0.1	1.61480	1.00372	0.89280	0.79128	1.02888	0.92753
±0.2	2.23229	0.96175	0.82584	1.05918	0.98639	0.86633
±0.3	2.77443	0.93750	0.78178	1.18822	0.96184	0.82599
±0.4	3.28400	0.92078	0.74808	1.49850	0.94460	0.79518
±0.5	3.77813	0.90821	0.72040	1.69769	0.93147	0.76997
±0.6	4.26639	0.89827	0.69669	1.88969	0.92096	0.74848
±0.7	4.75525	0.89012	0.67576	2.07687	0.91229	0.72964
±0.8	5.24980	0.88328	0.65691	2.26078	0.90494	0.71280
±0.9	5.75448	0.87741	0.63965	2.44255	0.89861	0.69754
±1.0	6.27354	0.87231	0.62364	2.62304	0.89308	0.68353
	$C=-10$ dB			$C=-20$ dB		
R(dB)	h	α	β	h	α	β
±0.1	0.42504	1.03858	0.94222	0.12382	1.04405	0.94959
±0.2	0.56125	0.99753	0.88351	0.16238	1.0037	0.89214
±0.3	0.67654	0.97297	0.84474	0.19486	0.97875	0.85417
±0.4	0.78156	0.95561	0.81513	0.22434	0.96135	0.82516
±0.5	0.88032	0.94230	0.79090	0.25197	0.94797	0.80141
±0.6	0.97485	0.93160	0.77025	0.27834	0.93719	0.78118
±0.7	1.06637	0.92272	0.75217	0.30379	0.92822	0.76347
±0.8	1.15564	0.91518	0.73604	0.32855	0.92058	0.74767
±0.9	1.24320	0.90866	0.72143	0.35276	0.91397	0.73337
±1.0	1.21947	0.90294	0.70805	0.37654	0.90816	0.72028

TABLE II

NORMALIZED EVEN-MODE IMPEDANCES FOR FIVE-SECTION COUPLERS OF VARIOUS DEGREES OF COUPLING

	$C=-3$ dB			$C=-6$ dB		
R(dB)	$Z_1=Z_5$	$Z_2=Z_4$	Z_3	Z_1	Z_2	Z_3
±0.1	1.078699	1.373506	3.986466	1.045012	1.219723	2.381814
±0.2	1.109498	1.441344	4.221770	1.060520	1.253022	2.460101
±0.3	1.136973	1.496760	4.413494	1.073916	1.279190	2.520679
±0.4	1.163141	1.546841	4.588548	1.086326	1.302028	2.573317
±0.5	1.188797	1.594271	4.757218	1.098178	1.322937	2.621589
±0.6	1.214383	1.640454	4.925031	1.109689	1.342617	2.667275
±0.7	1.240192	1.686272	5.095649	1.120989	1.361476	2.711417
±0.8	1.266456	1.732366	5.271895	1.132166	1.379781	2.754697
±0.9	1.293375	1.779253	5.456244	1.143280	1.397715	2.797598
±1.0	1.321138	1.827391	5.651062	1.154380	1.415418	2.840483
	$C=-10$ dB			$C=-20$ dB		
R(dB)	Z_1	Z_2	Z_3	Z_1	Z_2	Z_3
±0.1	1.025804	1.125813	1.677964	1.007738	1.037071	1.172210
±0.2	1.034177	1.143162	1.709224	1.010159	1.041832	1.178726
±0.3	1.041309	1.156560	1.732889	1.012202	1.045462	1.183583
±0.4	1.047843	1.618082	1.753050	1.014060	1.048550	1.187666
±0.5	1.054018	1.178486	1.771193	1.015805	1.051313	1.191295
±0.6	1.059959	1.188152	1.788048	1.017473	1.053857	1.194628
±0.7	1.065735	1.197297	1.804038	1.019085	1.056244	1.197755
±0.8	1.071395	1.206060	1.819432	1.020655	1.058513	1.200733
±0.9	1.076971	1.214539	1.834415	1.022193	1.060691	1.203599
±1.0	1.082485	1.222802	1.849121	1.023706	1.062798	1.206383

Note: Z_1, Z_2, Z_3, Z_4, Z_5 are the normalized even-mode impedances.

aid of a computer. Assuming that the denominator roots are known, the expression for $|\Gamma(t)|^2$ takes the form

$$|\Gamma(t)|^2 = K^2 \frac{t^2(t^2-t_1^2)^2(t^2-t_1^{*2})^2}{(t^2-t_1'^2)(t^2-t_2'^2)(t^2-t_2'^{*2})(t^2-t_3'^{*2})(t^2-t_3'^2)} \tag{54}$$

where $K^2=k_1^2/(1+k_1^2)$.

Consideration of previously stated realization conditions leads to

$$\Gamma(t) = K \frac{t(t^2 - t_1^2)(t^2 - t_1^{*2})}{(t + t_1')(t + t_2')(t + t_2'^*)(t + t_3')(t + t_3'^*)} \tag{55}$$

where the real parts of t_r' are all positive. Note that the numerator polynomial is odd in t, which is necessary if the resulting network is to be symetrical [9]. After the denominator roots of $\Gamma(t)$ are determined, the synthesis procedure is similar to that used for the three-section coupler. This procedure was programmed on a computer and carried out for a mean coupling of -3, -6, -10, and -20 dB. The results are presented in Table II. Figure 3 shows the bandwidth of operation vs. coupling deviation. It is interesting to note that our Table II and Tables A-6, 7, 8, which were independently compiled by Cristal and Young [3], contain identical results. Furthermore, the coupling response of symmetrical five-section -3-dB, ± 1, ± 2, ± 3, ± 4, ± 5-dB couplers was computed and found to be identical to their graph of Fig. 2.

From Fig. 3 it is seen that the -3-dB five-section coupler has an extremely wide bandwidth of operation. For example, a bandwidth of 9.6:1 can be obtained for a maximum coupling deviation of ± 0.5 dB as compared with 5.8:1 for a three-section coupler. Realization of the theoretical bandwidth of 9.6:1 for a five-section coupler, in practice, will depend upon the establishment of constructional techniques which will alleviate the severe discontinuity effects; the re-entrant coupled cross section introduced by Cohn [8] should prove advantageous in this case.

Experimental Five-Section Directional Coupler

A photograph of an experimental -10-dB strip-line directional coupler appears in Fig. 5. This directional coupler was designed to operate over a 7.21:1 frequency band centered at 2.278 Gc with a ± 0.5-dB coupling deviation around -10 dB. The input impedance of this coupler was chosen to be $Z_0=50$ ohms, and the even- and odd-mode characteristic impedances were obtained from Table II. By means of published formulas [10], the essential dimensions were computed in terms of the given Z_{oei} and Z_{ooi} values. In the coupled region, the plate spacing is 0.500 inch and the strip thickness is 0.200 inch. The 50-ohm coaxial input lines have an ID of 0.170 inch and an OD of 0.391-inch conductors supported by undercut teflon beads.

Preliminary measurements revealed good agreement with the theoretical coupling response, but poor directivity, having a minimum of 13 dB at 4.0 kMc. This may be attributed to discontinuity effects at the junctions between the sections. Internal reflections resulting from the steps between the coupler sections were partially canceled by the placement of ridges on the ground planes near these points. These matching structures are visible in Fig. 5. The coupling response in Fig. 6 shows very good agreement between theory and experiment. The directivity is greater than 18 dB over the 7.2:1 band from 0.555 to 4.0 Gc. A higher directivity response could certainly be obtained by further work on the various discontinuities. Figure 7 shows the

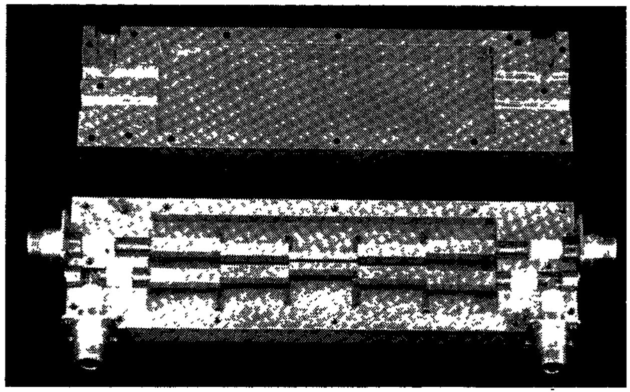

Fig. 5. Photograph of a five-section coupler for a 555–4000-Mc band. (Dimensions: 9.575 by 2.700 by 0.500 inches.)

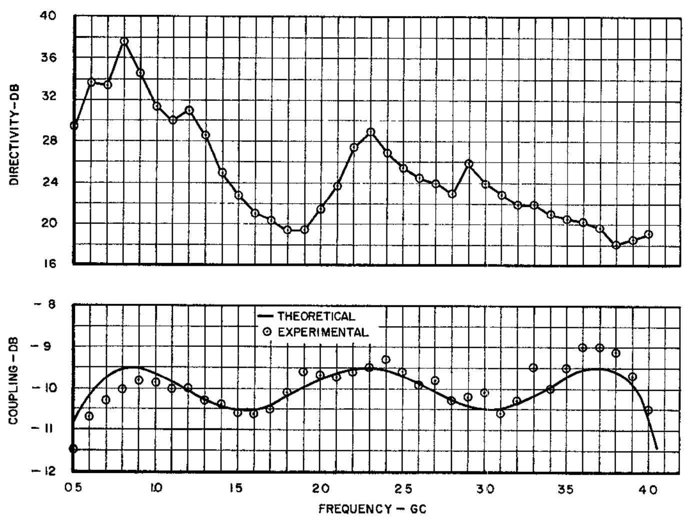

Fig. 6. Response of a 555–4000-Mc five-section 10-dB coupler.

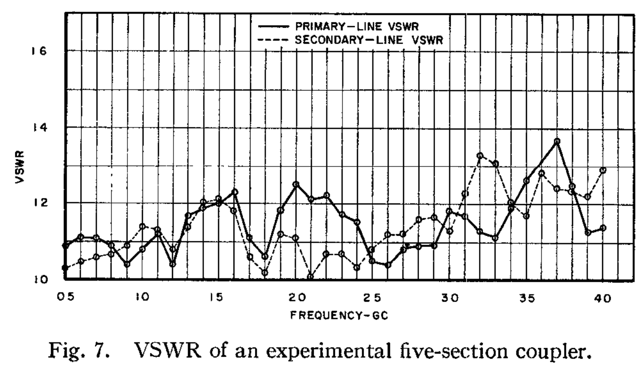

Fig. 7. VSWR of an experimental five-section coupler.

measured VSWR of both primary and secondary lines over the frequency band.

Conclusions

General synthesis procedures have been established for three-section and five-section symmetrical TEM-mode directional couplers. The synthesis leads to explicit formulas for the essential parameters, i.e., the normalized even-and odd-mode impedances, of three-section couplers. Although explicit formulas for the five-section couplers are not so readily obtainable, a sufficient amount of design information (in table form) is given for most practical coupler designs. An experimental model of a five-section coupler was built and tested, giving excellent agreement with theory.

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Theory and Tables of Optimum Symmetrical TEM-Mode Coupled-Transmission-Line Directional Couplers

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Abstract—**New equal-ripple polynomials were determined and applied to the synthesis of symmetrical TEM-mode coupled-transmission-line directional couplers (using exact methods). Tables of designs for symmetrical couplers of three, five, seven, and nine sections having mean couplings of −3.01, −6, −8.34, −10, and −20 dB, and having several equal-ripple tolerances in the coupling band are presented. Symmetrical maximally-flat directional-coupler designs having three, five, seven, and nine sections are also presented to complete the tables.**

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I. Introduction

A. General Properties of the Couplers

A SYMMETRICAL TEM-mode coupled-transmission-line directional coupler is shown schematically in Fig. 1. Note that the symmetrical directional coupler has symmetry with respect to two planes: Ports 1 and 2 have end-to-end symmetry with respect to Ports 3 and 4; Ports 2 and 3 have side-to-side symmetry with respect to Ports 1 and 4.

A TEM-mode coupled-transmission-line directional coupler, whether symmetrical or not, has the following properties, when a signal generator is connected to Port 1:

1) There is transfer of power from Port 1 to Port 2.
2) There is transfer of power from Port 1 to Port 4.
3) There is no transfer of power from Port 1 to Port 3.
4) There is no reflected wave out of Port 1.